

Quantum state transfer in optomechanical arrays

G. D. de Moraes Neto,^{1,*} F. M. Andrade,^{1,2,3,†} V. Montenegro,^{1,‡} and S. Bose^{1,§}

¹*Department of Physics and Astronomy, University College London, London WC1E 6BT, United Kingdom*

²*Department of Computer Science, University College London, London WC1E 6BT, United Kingdom*

³*Departamento de Matemática e Estatística, Universidade Estadual de Ponta Grossa, 84030-900 Ponta Grossa-PR, Brazil*

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Quantum state transfer between distant nodes is at the heart of quantum processing and quantum networking. Stimulated by this, we propose a scheme where one can highly achieve quantum state transfer between sites in a cavity quantum optomechanical network. There, each individual cell site is composed of a localized mechanical mode which interacts with a laser-driven cavity mode via radiation pressure, and photons exchange between neighboring sites is allowed. After the diagonalization of the Hamiltonian of each cell, we show that the system can be reduced to an effective Hamiltonian of two decoupled bosonic chains, and therefore we can apply the well-known results regarding quantum state transfer in conjunction with an additional condition on the transfer times. In fact, we show that our transfer protocol works for any arbitrary quantum state, a result that we will illustrate within the red sideband regime. Finally, in order to give a more realistic scenario we take into account the effects of independent thermal reservoirs for each site. Thus, solving the standard master equation within the Born-Markov approximation, we reassure both the effective model as well as the feasibility of our protocol.

I. INTRODUCTION

For quantum information processing purposes one often needs to transfer a quantum state from one site to another [1], this corresponding to the central goal in quantum networking schemes. A wide range of physical systems able to carry information are used for this end. For instance, proposals for quantum logical processing using trapped atoms, for example, making use of traveling photons to transfer states in cavity quantum electrodynamics (QED) [2] and phonons in ion traps [3].

Although photons and phonons are individual quantum carriers on themselves, several promising technologies for the implementation of quantum information processing rely on collective phenomena to transfer quantum states, such as optical lattices [4] and arrays of quantum dots [5] just to name a few. It is therefore, a main goal to find physical systems that provide robust quantum data bus (QDB) linking different quantum processors.

In recent years, extensive theoretical research have been carried out on the topic of state transfer in quantum networks, and many of them have been conducted in several different systems and architectures [6].

Interestingly, a plethora of results have been obtained based on qubit-state transfer through spin chains considering different types of neighbor (site-site) couplings [7, 8], as well as errors and detrimental effects arising from network imperfections/non-idealities [9–11].

On the other hand, optical lattices constitute a promising platform for quantum information processing, where both the coherent transport of atomic wave packets [12]

as well as the evolution of macroscopically entangled states [13] have been achieved.

Furthermore, significant advances have been made in engineered (passive) quantum networks, where the adjustment of static parameters leads to quantum information tasks, such as, entanglement generation and state transfer [14].

Motivated for all these aforementioned quantum systems towards quantum networking/processing, we present the state transfer of quantum information in optomechanical cavity systems — a promising growing field, where “weak” light-matter interactions (trilinear radiation pressure interaction) take place leading to interesting quantum effects [15].

Specifically, we show that information encoded on polariton states, *i.e.*, photonic-phononic combined excitations, can be used to transfer information from one site to another. Additionally, the use of polariton states allow us to link both the degrees of freedom of the quantized electromagnetic radiation field as well as the mechanical mode. Furthermore, polaritons permit undemanding manipulations with an external laser field. In fact, quantum state transfer of polaritonic qubits (photonic-atomic excitations) in a coupled cavity system have been demonstrated [16].

We would like to stress that, recent works on network of coupled optomechanical cells [17] and light storage [18] have been introduced. Also, collective effects as synchronization [19] quantum phase transitions [20] and generation of entanglement [21] have been proposed in the optomechanical field.

Moreover, in earlier studies of quantum state transfer in optomechanical systems relies on some sort of external control in the realm of active small networks [22, 23] or quantum state transfer only between mechanical modes [24]. The most straightforward approach in this context pertains to a sequence of SWAP gates, which ensure the

* gdmneto@gmail.com

† fmandrade@uepg.br

‡ v.montenegro.11@ucl.ac.uk

§ s.bose@ucl.ac.uk

successive transfer of the state between neighboring sites. While intuitively simple, active networks are considered to be very susceptible to errors —*which they are accumulated in each operation applied during the transfer*, as well as to dissipation and detrimental effects due to decoherence [6].

However, alternative strategies are based on the idea of eigenmode mediated state transfer and rely on a perturbative coupling and ensure resonance between the common frequency of the sender and the receiver and a single normal mode of the QDB [25] or a tunneling-like mechanism, described by a two-body Hamiltonian, which allows either a bosonic or a fermionic state to be transferred directly from the sender to the receiver, without populating the QDB [26].

In this manuscript, we envisage the quantum state transfer from a sender to a receiver in an array of optomechanical cells. There, each cell is composed of a localized mechanical mode that interacts with a laser-driven cavity mode via radiation pressure, and therefore photons can hop between neighboring sites.

In addition, we show how to design the parameters that allow us perfect state transfer of an arbitrary quantum state. In fact, two-way simultaneous communications for different pairs of sites without mutual interference. We stress that the linearization of the non-linear optomechanical Hamiltonian does not constitute a major restriction. For example, for driven optomechanical systems in the strong single-photon regime, we can both transfer information encoded in polariton states arising from ion trap-like Hamiltonian [27] as well as dark states in optomechanical systems [28].

Finally, we illustrate the effectiveness of our protocol when each cell is in contact with a thermal environment and under the red sideband regime.

II. THE MODEL

We consider a one-dimensional array of N optomechanical cells, each of these cells consists of a mechanical mode of angular frequency ω_m^n coupled via radiation pressure to a cavity mode of angular frequency ω_r^n . In addition, we consider an external laser driving the optical mode at angular frequency ω_p^n , as schematically depicted in Fig. 1(a).

Following the standard linearization procedure for driving optical modes in optomechanical cavities, we can recast the following Hamiltonian (in units of Planck constant, *i.e.*, $\hbar = 1$)

$$\hat{H}_n^L = -\Delta_p^n \hat{a}_n^\dagger \hat{a}_n + \omega_m^n \hat{b}_n^\dagger \hat{b}_n - G_n (\hat{b}_n + \hat{b}_n^\dagger) (\hat{a}_n + \hat{a}_n^\dagger), \quad (1)$$

where, the mechanical (optical) mode of the n -th cell is associated with the bosonic operator \hat{b}_n (\hat{a}_n); $\Delta_p^n = \omega_p^n - \omega_r^n$ is the pump detuning from cavity resonance, g_n corresponds to the single-photon coupling rate and $G_n = \alpha_n g_n$ is the effective optomechanical coupling strength proportional to the laser amplitude.

Here the cells are coupled by evanescent coupling between nearest neighbors cavities with hopping strength J_n , an interaction described by

$$\hat{H}_I = \sum_{n=1}^{N-1} J_n (\hat{a}_n^\dagger \hat{a}_{n+1} + \hat{a}_{n+1}^\dagger \hat{a}_n). \quad (2)$$

As seen from the above Hamiltonian \hat{H}_n^L (with $\Delta_p^n < 0$), we can readily notice two linearly coupled quantum harmonic oscillators. To obtain the relevant decoupled effective Hamiltonian, we proceed to diagonalization of the Hamiltonian using the usual Bogoliubov transformation as following:

$$\begin{aligned} \hat{A}_n &= \mathcal{N}_- \left[\Delta_1^n (\Omega_-^n) \hat{a}_n^\dagger + \Delta_2^n (\Omega_-^n) \hat{b}_n^\dagger \right. \\ &\quad \left. + \Delta_3^n (\Omega_-^n) \hat{a}_n + \Delta_4^n (\Omega_-^n) \hat{b}_n \right], \\ \hat{B}_n &= \mathcal{N}_+ \left[\Delta_1^n (\Omega_+^n) \hat{a}_n^\dagger + \Delta_2^n (\Omega_+^n) \hat{b}_n^\dagger \right. \\ &\quad \left. + \Delta_3^n (\Omega_+^n) \hat{a}_n + \Delta_4^n (\Omega_+^n) \hat{b}_n \right], \end{aligned} \quad (3)$$

with eigenvalues

$$\begin{aligned} (\Omega_{\mp}^n)^2 &= \frac{\Delta_p^{n2} + \omega_m^{n2}}{2} \\ &\quad \mp \frac{1}{2} \sqrt{((\Delta_p^n)^2 - (\omega_m^n)^2)^2 - 16G_n^2 \Delta_p^n \omega_m^n}, \end{aligned} \quad (4)$$

where we have defined

$$\begin{aligned} \Delta_1 (\Omega_{\mp}^n) &= 2G_n^2 \omega_m^n - (\Omega_{\mp} - \omega_m^n) (\Omega_{\mp}^n - |\Delta_p^n|) (\Omega_{\mp}^n + \omega_m^n), \\ \Delta_2 (\Omega_{\mp}^n) &= G_n [(\Omega_{\mp} - |\Delta_p^n|) (\Omega_{\mp} - \omega_m^n)], \\ \Delta_3 (\Omega_{\mp}^n) &= 2G_n^2 \omega_m^n, \\ \Delta_4 (\Omega_{\mp}^n) &= G_n (\Omega_{\mp} - |\Delta_p^n|) (\Omega_{\mp} + \omega_m^n), \end{aligned}$$

and normalization

$$\begin{aligned} \frac{1}{\mathcal{N}_{n,\mp}^2} &= [\Delta_3 (\Omega_{\mp}^n)]^2 + [\Delta_4 (\Omega_{\mp}^n)]^2 \\ &\quad - [\Delta_1 (\Omega_{\mp}^n)]^2 - [\Delta_2 (\Omega_{\mp}^n)]^2. \end{aligned} \quad (5)$$

Therefore, the total Hamiltonian $\hat{H} = \hat{H}_n^L + \hat{H}_I$ in the polariton basis can be rewrite as

$$\begin{aligned} \tilde{H} &= \sum_{n=1}^N \Omega_n^n \hat{A}_n^\dagger \hat{A}_n + \Omega_+^n \hat{B}_n^\dagger \hat{B}_n \\ &\quad + \sum_{n=1}^{N-1} (\lambda_n \hat{A}_n^\dagger \hat{A}_{n+1} + \zeta_n \hat{B}_n^\dagger \hat{B}_{n+1} + \text{H.c.}), \end{aligned} \quad (6)$$

with the effective tunneling strength

$$\begin{aligned} \lambda_n &= J_n \mathcal{N}_{n,-} \mathcal{N}_{n+1,-} [\Delta_1 (\Omega_-^n) \Delta_1 (\Omega_-^{n+1}) \\ &\quad + \Delta_3 (\Omega_-^n) \Delta_3 (\Omega_-^{n+1})] \end{aligned}$$

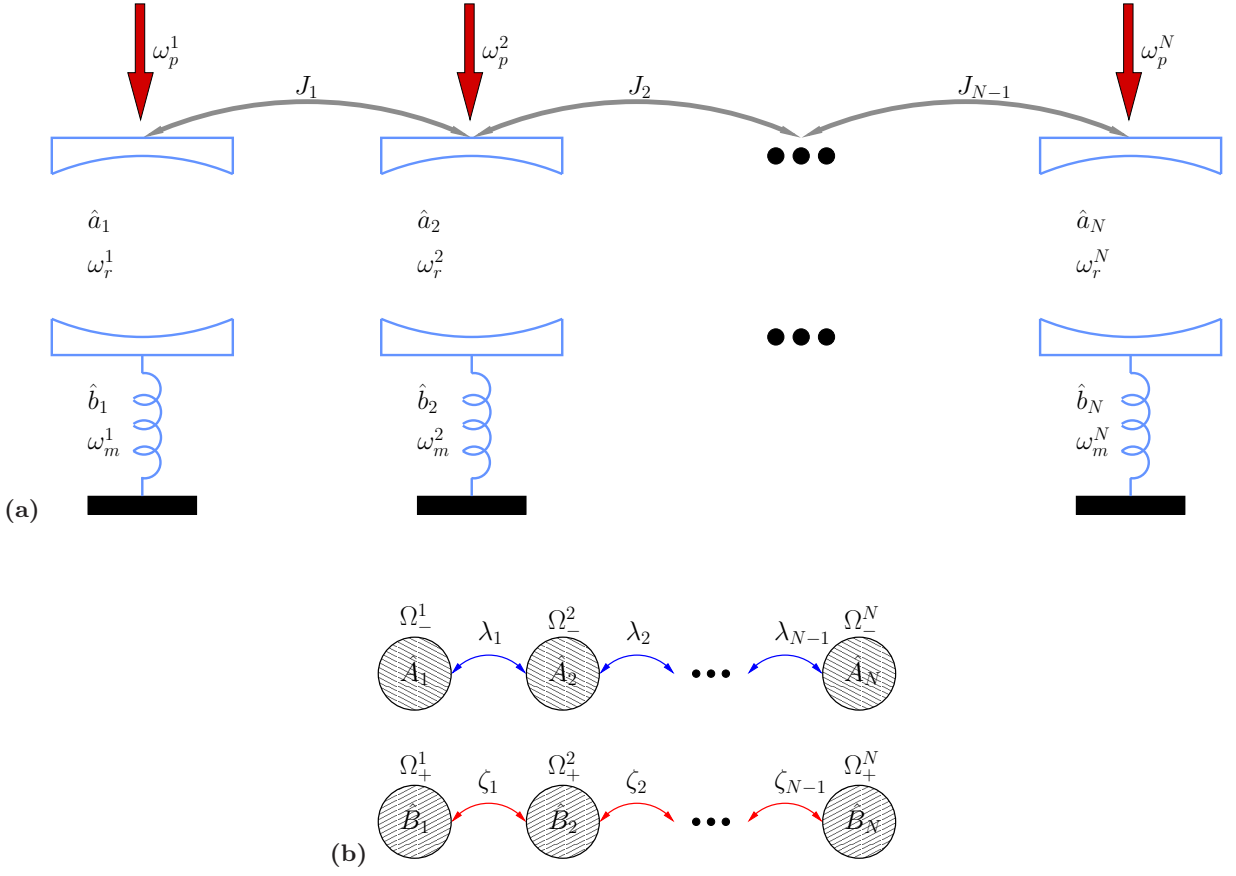


FIG. 1. (Color online) (a) Sketch of an array of N optomechanical cells, each of these cells consists of a mechanical mode of frequency ω_m^n coupled via radiation pressure to a cavity mode of frequency ω_r^n . The optical mode is driven by a laser at frequency ω_p^n and cells are coupled by evanescent coupling between nearest neighbors cavities with hopping strength J_n . (b) Schematic of the effective model, two decoupled bosonic chains with polaritonic energies Ω_-^n, Ω_+^n and neighbor-site hopping λ_n and ζ_n respectively.

and

$$\zeta_n = J_n \mathcal{N}_{n,+} \mathcal{N}_{n+1,+} [\Delta_2(\Omega_+^n) \Delta_2(\Omega_+^{n+1}) + \Delta_4(\Omega_+^n) \Delta_4(\Omega_+^{n+1})].$$

It is important to point out that in deriving the above expression, terms like $A_i^\dagger A_{i+1}^\dagger$ and $A_i^\dagger B_{i+1}$ have been neglected due to the usual rotating-wave approximation (RWA), which remains valid for

$$|\Omega_+^n - \Omega_-^n| \gg \sqrt{\sum_n^N \langle \hat{a}_n^\dagger \hat{a}_n \rangle + \langle \hat{b}_n^\dagger \hat{b}_n \rangle (\lambda_n + \zeta_n)}.$$

Now, it is straightforward to observe under the above mapping that the original full Hamiltonian of a unidimensional array of optomechanical cells becomes equivalent to a Hamiltonian of two distinct bosonic chains, this Hamiltonian being the central result of this manuscript. Scenario schematically illustrated in Fig. 1(b). Because of the effective structure achieved above, *i.e.*, two independent chains, we are now in position to take advantage of the well-known results on quantum state transfer.

As known from any state transfer scheme, the set of couplings parameters $\{\lambda_n, \zeta_n\}$ as well as energies Ω_\pm^n defines the transfer time τ . Here, we point out that our protocol requires that the transfer time for both polaritons $\hat{A}^\dagger \hat{A}$ and $\hat{B}^\dagger \hat{B}$ has to be the same or at least an odd multiple of each other.

To illustrate this point, we will consider the red-detuned regime $\Delta_p^n \approx -\omega_m^n$, thus the Hamiltonian (1) can be simplified as

$$\hat{H}_n^{\text{red}} = \omega_m^n (\hat{a}_n^\dagger \hat{a}_n + \hat{b}_n^\dagger \hat{b}_n) - G_n (\hat{b}_n \hat{a}_n^\dagger + \hat{b}_n^\dagger \hat{a}_n). \quad (7)$$

To obtain the diagonal form of the above expression, we consider the operators

$$\hat{A}_n = \frac{(\hat{a}_n + \hat{b}_n)}{\sqrt{2}}, \quad \hat{B}_n = \frac{(\hat{a}_n - \hat{b}_n)}{\sqrt{2}} \quad (8)$$

with eigenvalues $\omega_A^n = \omega_m^n - G_n$ and $\omega_B^n = \omega_m^n + G_n$, respectively.

For the strongly off-resonant regime ($G_n \gg J_n$) together with the RWA, we can recast the following polari-

ton Hamiltonian

$$\hat{H} = \sum_{n=1}^N \left(\omega_A^n \hat{A}_n^\dagger \hat{A}_n + \omega_B^n \hat{B}_n^\dagger \hat{B}_n \right) + \sum_{i=1}^{N-1} \frac{J_n}{\sqrt{2}} \left(\hat{A}_n^\dagger \hat{A}_{n+1} + \hat{B}_n^\dagger \hat{B}_{n+1} + \text{H.c.} \right). \quad (9)$$

Now we proceed to choose a set of parameters that allows quantum state transfer. For instance, a straightforward set can be found in Ref. [6] corresponding to $\omega_m^n = \omega_m$, $G_n = G$ and $J_n = (J/\sqrt{2})\sqrt{n(N-n)}$, which provides the same transfer time for each chain $\tau_A = \tau_B = \pi/J$.

Therefore, regardless a relative phase depending on ω_A and ω_B which is fixed and known, and hence, it can be amended, any optomechanical state can be transferred only ensuring the $G \gg J$ regime together with $J_n = (J/\sqrt{2})\sqrt{n(N-n)}$.

However, we stress that any other protocol could have been chosen for this purpose. For example, schemes based on eigenmodes, where one of many possibilities that permit quantum state transfer is the following set of parameters: $J_1 = J_{N-1} = \lambda \ll J_k = J \ll G$, $k = 2 \dots N-2$, N being an odd number and $\omega_m^n = \omega_m$, $G_n = G$.

On the other hand, on resonant schemes [25] the shorter transfer time possible corresponds to $\tau_A = \tau_B = (\pi/\lambda)\sqrt{2(N+1)}$, and for tunneling-like protocol [26] with same parameters and conditions $\omega_m^1 = \omega_m^N = \omega_m + \delta$ and $\lambda \ll \delta \ll J$, we obtain transfer times $\tau_A = \tau_B = N\pi\delta/2\lambda^2$.

Finally, it is worth stressing that, the effect of a phononic hop term between neighboring sites only change the strength of λ_n and ζ_n .

III. DISSIPATIVE MECHANISMS

In this section, in a step towards a more realistic model we take into account decoherence and dissipation. To fulfill this goal, we employ the standard formalism for open quantum systems, *i.e.*, we solve the dynamics of the optomechanical array using the master equation in Lindblad form within the Born-Markov approximation.

Furthermore, we numerically investigate the effectiveness of our model computing the fidelity for the state transfer considering engineered hop couplings between cells where each cell is considered in the red-sideband regime.

The master equation for the composite coupled system

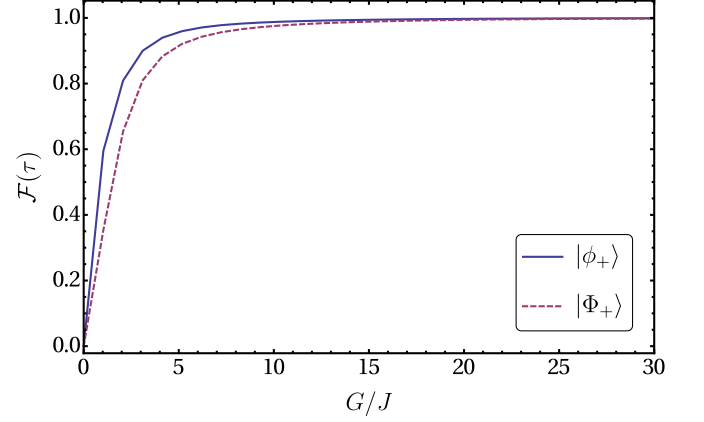


FIG. 2. (Color online) The figure shows the dynamics of the transfer fidelity at time $\tau = \pi/J$ as a function of G/J . The states $|\phi_+\rangle = (1/\sqrt{2})(|1, 0\rangle + |0, 1\rangle)$ and $|\Phi_+\rangle = (1/\sqrt{2})(|2, 0\rangle + |0, 2\rangle)$ corresponds to the initial states of the sender.

is given as

$$\frac{d\hat{\rho}}{dt} = -i [\hat{H}, \hat{\rho}] \quad (10)$$

$$+ \sum_{n=1}^N \frac{\kappa_n}{2} (1 + \bar{n}_c) \mathcal{D}[\hat{a}_n] \hat{\rho} + \frac{\kappa_n}{2} \bar{n}_c \mathcal{D}[\hat{a}_n^\dagger] \hat{\rho} + \frac{\gamma_n}{2} (1 + \bar{n}_m) \mathcal{D}[\hat{b}_n] \hat{\rho} + \frac{\gamma_n}{2} \bar{n}_m \mathcal{D}[\hat{b}_n^\dagger] \hat{\rho}, \quad (11)$$

where $\hat{H} = \sum_{n=1}^N \hat{H}_n^{\text{red}} + \hat{H}_I$ and the Lindblad term

$$\mathcal{D}[\hat{O}] = 2\hat{O}\hat{\rho}\hat{O}^\dagger - \hat{\rho}\hat{O}^\dagger\hat{O} - \hat{O}^\dagger\hat{O}\hat{\rho} \quad (12)$$

takes into account the dissipative mechanisms of the optics (mechanics) in contact with a thermal reservoir with occupation number \bar{n}_c (\bar{n}_m), where the photon (phonon) decay rate is given by κ_n (γ_n).

Needless to say that the first non-trivial quantum network in passive schemes is composed of four sites. Hence, for computational time purposes, we will exemplify our findings considering an array of four cells where the coupling fulfill $J_n = (J/\sqrt{2})\sqrt{n(N-n)}$ and $\omega_m^n = \omega_m$.

To validate the polariton Hamiltonian (9), we present the closed evolution of the transfer fidelity at time $\tau = \pi/J$ as a function of G/J , see Fig. (2).

In order to compute the fidelity of the transferred quantum state, we solved the closed quantum system dynamics (running the simulation in QuTiP [30]) considering the sender initially in the state $|\phi_+\rangle = (1/\sqrt{2})(|1, 0\rangle + |0, 1\rangle)$ or $|\Phi_+\rangle = (1/\sqrt{2})(|2, 0\rangle + |0, 2\rangle)$ (we have used the following notation $|a, b\rangle = |a\rangle_{\text{optics}} \otimes |b\rangle_{\text{mechanics}}$), where all the other cells are in the vacuum state.

Moreover, for our illustrative red sideband detuning regime ($-\Delta_p \approx \omega_m \gg \gamma, \kappa$) a well-known stability condition [31] given by $G < (1/2)\sqrt{\omega_m^2 + (\gamma^2 + \kappa^2)/4}$ come

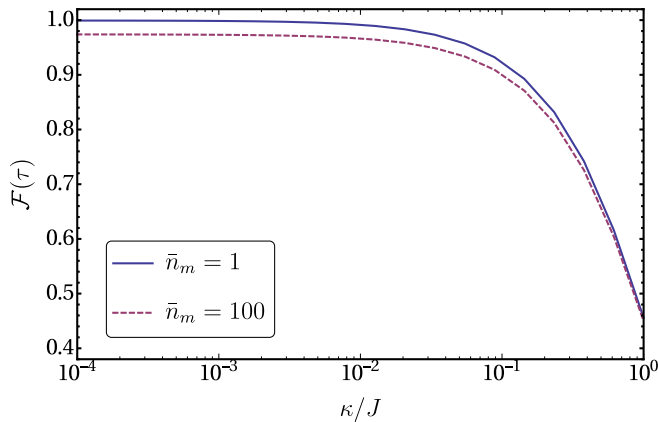


FIG. 3. (Color online) We illustrate the fidelity of the transfer process ($\tau = \pi/J$) as a function of κ/J (log-axis) for two mechanical phonon bath occupation number $\bar{n}_m = 100$ and $\bar{n}_m = 1$. To exhibit our findings, we consider the following feasible parameters in optomechanical crystals [29] in the microwave regime; $\omega_m/2\pi = 3.68 \times 10^9$, $\gamma/2\pi = 35 \times 10^3$, $\bar{n}_c = 0.005$, $G = (J/4) \times 10^2 = 5 \times 10^9$.

into sight, and therefore it must be observed throughout the quantum state transfer protocol. On the other hand, in order to achieve a fidelity value close to the unity, G has to be $G \approx (J/4) \times 10^2$ (as seen in Fig. 2). The effect of both the stability condition (being an upper bound for G), as well as the effectiveness of the fidelity ($\mathcal{F}(\tau) \rightarrow 1$), have as a result the limitation of the maximum coupling strength $J_{n=N/2} = NJ/4$ and consequently the maximum number of cells.

In Fig. 3, we compute the fidelity for the transfer of an initial quantum state given by $|\phi_+\rangle$ as a function of κ/J for two different mechanical phonon bath occupation number $\bar{n}_m = 100$ and $\bar{n}_m = 1$, where we have used the following currently experimental parameters in optomechanical crystals [29] in the GHz regime; $\omega_m/2\pi = 3.68 \times 10^9$, $\gamma/2\pi = 35 \times 10^3$, $\bar{n}_c = 0.005$, $G = (J/4) \times 10^2 = 5 \times 10^9$. The high fidelity shown in Fig. 3 up to $J = 10\kappa$ is an expected result, since $k_B T < \hbar\omega_m$, and the threshold for coherent operations take place when $\max(J_n) = \max(\gamma_n, \kappa_n)$. Thus, to achieve transfer fidelities close to unity for an array with $N = 100$ cells (with the same set of parameters considered above), we can then estimate the cavity linewidth as $\kappa \sim 10^5$.

Finally, we point out that the hopping coupling reported in [19] is in the range of THz. Hence, to achieve

the inequality $J < G$ within the stability region, we should engineered optomechanical arrays with larger lattice spacing and/or mechanical modes with frequencies above THz, being this last a challenging experimental scenario.

IV. CONCLUSION

We have thus advanced a theoretical proposal for quantum state transfer in optomechanical arrays. Our proposal relies on a general scheme illustrated by polariton transformation of the linearized Hamiltonian (6) that allow us to obtain an effective Hamiltonian of two decoupled bosonic networks.

The central result of the present manuscript is the derivation of the polariton Hamiltonian (6), where we can bring previous results from quantum state transfer protocols in bosonic networks. Specifically, we can apply any type of quantum state transfer scheme with an extra additional condition, namely, that the rate between the transfer times of both decoupled polaritonic chains must be an odd number. Furthermore, we analyze the effects of dissipation and a possible experimental implementation of our proposal in the red-sideband regime with experimental accessible parameters.

It is also important to point out that —*although not reported explicitly in this work*— the linearization of the non-linear optomechanical Hamiltonian does not constitute a major restriction. For instance, for driven optomechanical systems in the strong single-photon regime, we can both transfer information encoded in polariton states arising from ion trap-like Hamiltonian [27] as well as dark states in optomechanical systems [28].

Moreover, even though we used a one-dimensional array in this work, any other topology might be consider, such as lattices (2D) or crystals (3D) setups.

In addition, other interesting aspects to study are the “pretty good state transfer” schemes in Ref. [6], and the generation of long distance quantum entanglement between sites [26].

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